Most traders consider market data to be a continuous function. It is further assumed that smoothing this function with averages or squiggly line indicators will create patterns or conditions that are useful for predicting future market direction. These assumptions are very wrong on many levels.

First of all, market data is a nonstationary random process. Basically this means that whatever pattern you are observing, you will never see that pattern again. The process can be formulated as the classic Drunkards Walk problem. The results are partial differential equations called the Diffusion Equation or the Telegraphers Equation, depending on the selection of the random variable. There is no direct solution to these equations because they are boundary value problems, and the boundaries cannot be defined for market data.

Secondly, market data has a pink noise spectrum. Just like light waves, the term spectrum means that the data are comprised of cycle components having a wide range of frequencies, or periods. The colors of the spectrum comprise white light. The pink noise spectrum of market data means that the longer cycle periods have a greater amplitude. Without arguing about details, we can use the assumption that market data cycle amplitudes are in direct proportion to their cycle periods as a workable hypothesis. For example, if you look at a chart of daily data it is not easily discernable from a chart of weekly data if you remove the labels. This being the case, since the horizontal time scale varies 5:1, then it necessarily follows that the vertical amplitude scale must also vary in the ratio 5:1. Therefore cycle amplitudes statistically increase at the rate of 6 dB per octave. It must be noted that noise does not necessarily mean chaos. The cycles in market data can very well contain information. For example, pink noise implies memory in the data.

Most of all, since the data is comprised of cycle components, the question is whether these components combine in the form of AM (amplitude modulation) or FM (frequency modulation). The modulation characteristics of market data can be determined by direct measurement.

MARKET DATA SPECTRUM

A high resolution estimate of the spectrum of market data can be made using the MESA (Maximum Entropy Spectral Analysis) program. The MESA display of the spectrum of SPY over the last year and a half is shown in Figure 1. The spectrum display is a heatmap

---

1 https://www.investopedia.com/articles/trading/07/stationary.asp
2 http://www.mesasoftware.com/ehlers_cycles_tutorial.htm
3 https://en.wikipedia.org/wiki/Pink_noise
that is synchronous with the candlestick chart above it. The period of the spectrum is scaled from a 12 bar cycle period to a 54 cycle period in the vertical dimension. The intensity of the spectral components range from white hot, through red hot, to ice cold in black. A single fixed period cycle would be depicted as a horizontal yellow line.

Clearly, market data does not contain a fixed cycle. There is a strong tendency for an approximately monthly 20 bar cycle period to be the dominant cycle. However, there are other significant components, some appearing simultaneously with the monthly cycle. There is also a strong tendency for the measured dominant cycle period to change rapidly from bar to bar, analogous to a “chirp” in the audio range.

![Figure 1. MESA Spectrum for SPY](image)

Even with all the variations of a nonstationary random process the dominant cycles are easily identified by observation. Therefore, MESA has identified market data as a narrow band random process. A narrow band process is one where the width of a region of the spectral density is small compared with the center period of that region. Such a process can be mathematically modelled in the form:

\[ S(t) = A(t) \cdot \text{Sine}(\omega t + \Theta(t)) \]

In this expression \( A(t) \) is a time-variable function amplitude modulating the Sine wave. In other words, it is Amplitude Modulating (AM) a carrier waveform. The constant angular frequency of the carrier waveform Sine wave is \( \omega \). \( \Theta(t) \) is a time variable phase term that is phase modulating the constant angular frequency, making the net frequency variable with time. Phase modulation is fundamentally the same as frequency modulation (FM) since frequency is just the rate-change of phase.
We continue our study of the market data structure by analyzing the AM and FM components of the narrow band nonstationary random process.

AMPLITUDE MODULATION COMPONENT ANALYSIS

We start our analysis by taking the one bar difference of the market data. This is analogous to taking the derivative of a continuous function in the calculus. This action has two major results; it places a zero in the transfer response and it whitens the pink noise spectrum. The zero is easy to understand because the value of the data is the same for the current bar and the previous bar at zero frequency. Now, consider moving away from zero frequency by a very, very small amount, e. Then the output of the difference can be expressed as:

\[
\text{Output} = (1 + e) - 1 = e
\]

In other words, the output of the difference grows in direct proportion to its distance from zero frequency. Reversing the direction, that means the amplitude falls off at the rate of 6 dB per octave. Since the transfer response of the difference falls off at the rate of 6 dB per octave and since the data intensity of the pink noise spectrum increases at the rate of 6 dB per octave, the net effect of taking the difference whitens the spectrum. That is, the spectrum is now effectively white noise, and we can therefore see the shorter wavelength components more clearly.

Figure 2 shows the signal of the whitened spectrum in the first subgraph. This is the classic picture of an Amplitude Modulated (AM) waveform. That is, the amplitude swings of both the positive and negative alternations are in proportion to the modulation waveform. In this case, an eyeball correlation allows me to assert that the AM is due to volatility. In fact, we can get a true measure of volatility by performing AM detection of this signal.

AM detection is done by rectifying the carrier and recovering the envelopes of the amplitudes of the resulting peak swings. In code, this is done by taking the absolute value of the waveform and estimating the envelope by the largest amplitude over the last few samples. With reference to Code Listing 1, the derivative is the Close – Open because this is basically the same as Close – Close[1], particularly for intraday data. Plus, it has the added benefit of automatically removing gap openings for intraday data. The phase information is stripped from the rectified waveform by using only the highest value over the last four bars. The resulting envelope is lightly smoothed to form the Volatility indicator.

The volatility indicator is shown in the lower subgraph in Figure 3. It compares favorably with an indicator constructed from the smoothed values of High – Low.
Figure 2. Signal of Whitened Spectrum is an Amplitude Modulated Waveform

Code Listing 1. AM Detector

```
{ 
    AM Detector  
    (C) 2020   John F. Ehlers  
}

Vars: 
    Deriv(0), 
    Envel(0), 
    Volatil(0);

Deriv = Close - Open;

Envel = Highest(AbsValue(Deriv), 4);

Volatil = Average(Envel, 8);

Plot1(Volatil);
Plot2(0);
```
FREQUENCY MODULATION COMPONENT ANALYSIS

Classical FM detection techniques are used to extract the frequency modulating, or phase modulating, components of the narrow band nonstationary random process. In addition to its other functions, the derivative is also a phase detector because it is a Finite Impulse Response (FIR) difference filter having a linear phase shift across the entire signal spectrum.

With reference to Code Listing 2, amplitude information is stripped from the whitened spectrum signal by running it through a hard limiter. The results of the hard limiter are shown in Figure 4.

The final step of creating a FM Demodulator indicator is to integrate the amplitude limited waveform in a SuperSmoother\(^4\) filter. Figure 5 shows that the FM Demodulator Indicator accurately tracks timing of price variations. For example, you can correlate major swings in the price chart with peaks and valleys of the indicator in the subgraph.

---

Figure 4. The Hard Limiter Removes All Amplitude Information from the Whitened Spectrum

Code Listing 2. FM Demodulator Indicator

```
{ 
    FM Demodulator Indicator
    (C) 2013-2020 John F. Ehlers
}

Inputs:
    Period(30);

Vars:
    Deriv(0), HL(0),
    a1(0), b1(0), c1(0), c2(0), c3(0), SS(0);

//Derivative to establish zero mean (Basically the same as Close - Close[1], but removes intraday gap openings)
    Deriv = Close - Open;

//Hard limiter to remove AM noise
    HL = 10*Deriv;
    If HL > 1 Then HL = 1;
    If HL < -1 Then HL = -1;

//Integrate with a SuperSmoother
    a1 = expvalue(-1.414*3.14159 / Period);
```
b1 = 2*a1*Cosine(1.414*180 / Period);
c2 = b1;
c3 = -a1*a1;
c1 = 1 - c2 - c3;
SS = c1*(HL + HL[1]) / 2 + c2*SS[1] + c3*SS[2];
If Currentbar < 3 Then SS = Deriv;

Plot1(SS);
Plot2(0);

Figure 5. The FM Demodulator Indicator Accurately Tracks the Timing of Price Variations

CONCLUSIONS

 Entire books have been written about the pink noise spectral shape of market data. The fact that longer cycle periods have greater amplitude swings is an effect that I call Spectral Dilation. But, just because it is called noise, it does not mean that the cyclic components do not carry information. I showed by direct measurement using the MESA spectrum estimator that the data is a nonstationary narrow band random process, and therefore can be accurately modeled with AM and FM components. The AM components represent market volatility. The FM components contain market timing information.

Since many, if not most, technical indicators contain a mish-mash of AM and FM components and/or disregard the consequences of Spectral Dilation, these indicators are
distorted or give inaccurate timing signals. A careful review of the indicators you use is recommended.